

In questions where the definition of a Prolog predicate is asked, all predicates found in the textbook and on the slides (both built-in and locally defined) may be used freely. Please refer to individual subquestions with their numbers and letters (e.g., 2.b)!

1. Determine the outcome of the following Prolog queries (error, failure, success)! In case of success, specify the resulting variable substitutions! All queries are fed to the system independently. (Assume that the `lists` library is loaded.) (5 points)

- (a) $Z = 1+5$, $\backslash+$ $Z = 2*3$.
 (b) X is $4-3$, Y is $X+1$, $Y = 3-1$.
 (c) $D = 3+E$, $\backslash+$ $D = 2$, R is $D+1$.
 (d) `append([], [a,12|_], [A,_])`.
 (e) $2+4-2 = A-B$.

2. Write down the canonical form or draw the tree form of the both left and right hand sides of the following unifications. Specify the variable substitutions which the unifications lead to. (9 points)

- (a) $[X, [3*_]|Z] = .(Y, [[Y*2]])$.
 (b) $f(_+A*a, [C,_|B], F) = f(F+C, [3*_|b], 6)$.

3. Assume that the following program is loaded into the Prolog system.

```
p([A,B|_], T, E) :-
    A > T,
    A < B,
    E = A.
p([A|As], _, E) :-
    p(As, A, E).
```

Determine the values that X will take as a result of the following (independent) queries! Write down *all* solutions separated by semicolons, in the same order as the system would enumerate them! If there are no solutions, write `{no}`!

- (a) `p([2,4,1.2,3], 0, A)`.
 (b) `p([1,5,10], 2, A)`.
 (c) `p([3,4,1,5,8], 1, A)`.
 (d) `p([3,4,2,5,3,2], 4, A)`.
 (e) `p([2,4,6,7,8,2,4,5], 3, A)`.

Consider the following procedure, which uses the `p/3` predicate defined above:

```
% p(L, Z): Z is a member of the L list such that...
p(L, Z) :- L = [A|As], p(As, A, Z).
```

- (f) Describe in a declarative manner what this `p/2` predicate does by completing the above head comment. Make sure to specify the enumeration order of the solutions! (8 points)

4. Consider a list consisting of X - Y pairs. We call the pairs A - B and C - D *mergeable*, if either $A = C$ or $B = D$ holds, and they can be *merged* into the pair X - Y , where $X = A + C$ and $Y = B + D$. Write a Prolog procedure called `merged` which, given a list of such pairs as input in its first argument, **enumerates** the merged value of all mergeable neighboring list elements, preserving the order in which they appear in the list. Enumerate each merged value exactly once! If an auxiliary procedure is deemed necessary, write a declarative head comment for it! (8 points)

```
% merged(XYs, M): M is a merged value of two neighboring pairs in XYs.
| ?- merged([1-2], M).          ----> no
| ?- merged([1-2, 1-3], M).     ----> M = 2-5; no
| ?- merged([1-2, 1-2], M).     ----> M = 2-4; no
| ?- merged([1-2, 1-3, 4-3], M). ----> M = 2-5; M = 5-6; no
| ?- merged([1-2, 1-2, 1-2], M). ----> M = 2-4; M = 2-4; no
| ?- merged([1-2, 1-3, 3-5, 4-5], M). ----> M = 2-5; M = 7-10; no
```

